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Internal ring gears – design and considerations

When using internal ring gears, you can develop a gear system with a high reduction ratio in a compact space — but there are considerations regarding the reduction ratio, interference possibilities, and design window dimensions.

When one is under a great deal of stress, they have a choice to either internalize the stress or to express it. Some express it through exercise, some drown their sorrows at the neighborhood pub, and some choose to berate others. In gearing, there are situations when internalizing the gear mesh is desirable. For these applications, we specify ring gears.

Internal gears, ring gears, and internal ring gears are interchangeable terms for the same type of gear. These gears are composed of a cylindrical shape having teeth inside a circular ring. The gear teeth of an internal gear typically mesh with the teeth of a spur gear. Spur gears have a convex-shaped tooth profile and internal gears have reentrant shaped tooth profile; this characteristic is opposite of internal gears. The formulas for calculating the dimensions of internal gears and their interferences are quite different than those of other gearing.

Figure 1 presents the mesh of an internal gear and external gear. Of vital importance are the working pitch diameters (dw) and working pressure angle (w). They can be derived from center distance (a) and equations detailed below.

$$d_{w1} = 2a\frac{z_1}{z_2 - z_1}$$

$$d_{w2} = 2a\frac{z_2}{z_2 - z_1}$$

$$\alpha_w = \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a}\right)$$

Table 1 shows the formulas for calculating the geometry of a profile shifted internal gear and a non-shifted external gear. In this type of gear system, it is common for one or both members to be profile shifted in order to overcome the various interference fits.

If the center distance (a) is known, then x_1 and x_2 can be obtained from the inverse calculations of items 4 thru 8 of Table 1. These inverse formulas are detailed in Table 2.

There are three different types of interference can occur with internal gears: involute interference, trochoid interference, and trimming interference.

1. Involute interference (Figure 2) occurs when the distance between the dedendum of the external gear and the addendum of the internal gear is too narrow and the gears cannot mesh properly. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by observing the following cited conditions:

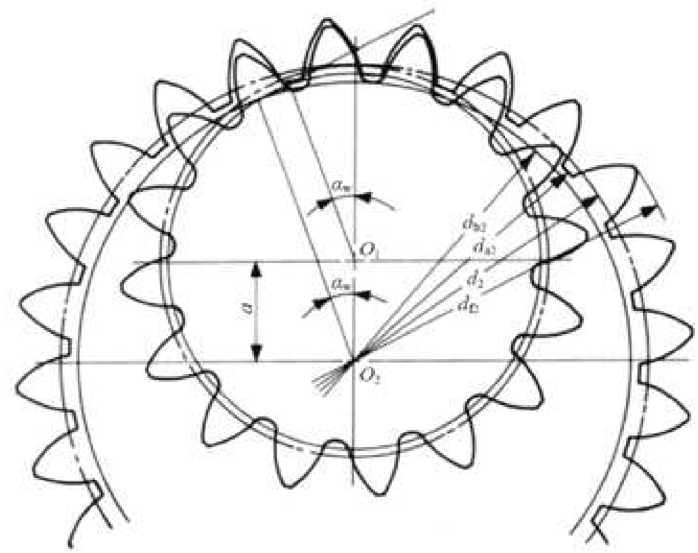

Figure 1: The meshing of internal gear and external gear.

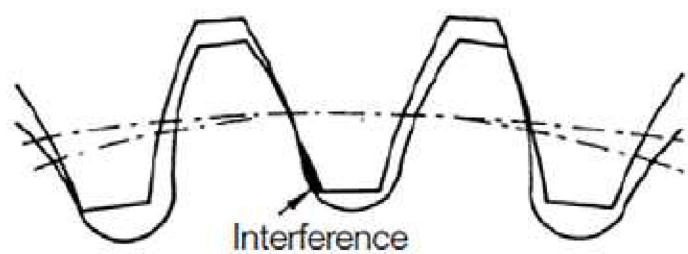

Figure 2: Involute interference.

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan \alpha_{a2}}{\tan \alpha_w}$$ **Equation 1**

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right)$$ **Equation 2**

$$\alpha_w = \cos^{-1}\left\{\frac{(z_2 - z_1)\, m \cos \alpha}{2a}\right\}$$ **Equation 3**

Equation 2 is true only if the tip diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geq d_{b2}$$ **Equation 4**

For a standard internal gear, where $\alpha = 20°$, Equation 4 is valid only if the number of teeth is $z_2 > 34$.

2. Trochoid interference refers to an interference occurring at the addendum of the external gear and at the dedendum of the internal gear during recess tooth action. This interference is due to the distance between the teeth being too shallow. It tends to happen when the difference between the numbers of teeth of the two gears is small.

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		16	24
4	Profile shift coefficient	x		0	+ 0.516
5	Involute function α_w	inv α_w	$2\tan\alpha\left(\frac{x_2 - x_1}{z_2 - z_1}\right) + \text{inv}\,\alpha$	0.061857	
6	Working pressure angle	α_w	Find from involute Function Table	31.321258°	
7	Center distance modification coefficient	y	$\frac{z_2 - z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.4000	
8	Center distance	a	$\left(\frac{z_2 - z_1}{2} + y\right) m$	13.2	
9	Reference diameter	d	zm	48.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	52.7998	79.1997
12	Addendum	h_{a1} h_{a2}	$(1 + x_1)m$ $(1 - x_2)m$	3.000	1.452
13	Tooth depth	h	$2.25m$	6.75	
14	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 - 2h_{a2}$	54.000	69.096
15	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{a2} + 2h$	40.500	82.596

Table 1: The calculations of a profile shifted internal gear and external gear where the module of the gears is 3, the number of teeth on the spur gear is 16, the number of teeth on the internal gear is 24, and the internal gear is profile shifted.

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Center distance	a		13.1683	
2	Center distance modification coefficient	y	$\frac{a}{m} - \frac{z_2 - z_1}{2}$	0.38943	
3	Working pressure angle	α_w	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_2 - z_1} + 1}\right)$	31.0937°	
4	Difference of profile shift coefficients	$x_2 - x_1$	$\frac{(z_2 - z_1)(\text{inv}\,\alpha_w - \text{inv}\,\alpha)}{2\tan\alpha}$	0.5	
5	Profile shift coefficient	x	—	0	0.5

Table 2: The calculations of the profile shift of an internal gear and external gear when the center distance is known.

The formulas for calculating the dimensions of internal gears and their interferences are quite different than those of other gearing.

Equation 5 presents the condition for avoiding trochoidal interference.

$$\theta_1 \frac{z_1}{z_2} \operatorname{inv} \alpha_w - \operatorname{inv} \alpha_{a2} \geq \theta_2$$

Equation 5

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}^2 - r_{a1}^2 - a^2}{2ar_{a1}}\right) \\ &\qquad + \operatorname{inv} \alpha_{a1} - \operatorname{inv} \alpha_w \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}^2 - r_{a1}^2}{2ar_{a2}}\right) \end{aligned}\right\}$$

Equation 6

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right)$$

Equation 7

In the meshing of an external gear and a standard internal gear where the pressure angle $\alpha = 20°$, trochoid interference is avoided if the difference of the number of teeth, $z_2 - z_1$, is larger than 9. (Figure 3)

3. Trimming interference occurs in the radial direction in that it prevents the pulling of the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. Equation 8 indicates how to prevent this type of interference.

$$\theta_1 + \operatorname{inv} \alpha_{a1} - \operatorname{inv} \alpha_w \geq \frac{z_2}{z_1}(\theta_2 + \operatorname{inv} \alpha_{a2} - \operatorname{inv} \alpha_w)$$

Equation 8

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1 - (\cos \alpha_{a1}/\cos \alpha_{a2})^2}{1 - (z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos \alpha_{a2}/\cos \alpha_{a1})^2 - 1}{(z_2/z_1)^2 - 1}} \end{aligned}\right\}$$

Equation 9

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling.

Table 3 shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle $\alpha_0 = 20°$, and no profile shift, i.e., $x_0 = 0$.

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_0 = 15 to 22).

Table 4 shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction (x_0) is the magnitude of shift, which was assumed to be: $x_0 = 0.0075 z_0 + 0.05$.

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_0 = 15 to 19).

When using internal ring gears, you can develop a gear system with a high reduction ratio in a compact space. However, there are many considerations regarding the reduction ratio, interference possibilities, and design window dimensions.

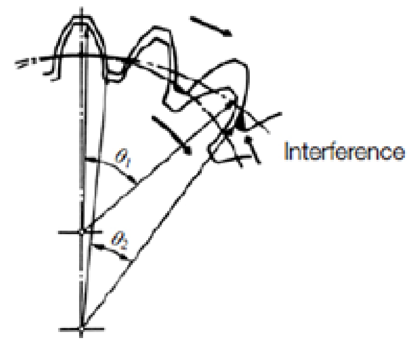

Figure 3: Trochoid interference.

$\alpha_0 = 20°, x_0 = x_2 = 0$

z_0	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45
z_0	28	30	31	32	33	34	35	38	40	42	
z_2	46	48	49	50	51	52	53	56	58	60	
z_0	44	48	50	56	60	64	66	80	96	100	
z_2	62	66	68	74	78	82	84	98	114	118	

Table 3: The limit to prevent an internal gear from trimming interference.

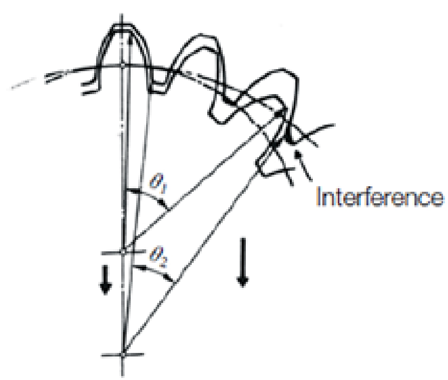

Figure 4: Trimming interference.

$\alpha_0 = 20°, x_2 = 0$

z_0	15	16	17	18	19	20	21	22	24	25	27
x_0	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50
z_0	28	30	31	32	33	34	35	38	40	42	
x_0	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	
z_2	52	54	55	56	58	59	60	64	66	68	
z_0	44	48	50	56	60	64	66	80	96	100	
x_0	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8	
z_2	71	76	78	86	90	95	98	115	136	141	

Table 4: The limit to prevent an internal gear from trimming interference.

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