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Determining tooth thickness of various gear types – Part II

How to calculate the nominal values of span measurement of teeth various types of gearing.

In order to determine the tooth size of a gear after taking into account the backlash allowance, you first must determine what the nominal tooth thickness should be. There are three methods for determining this value. They are chordal tooth thickness measurement, span measurement, and over pin or ball measurement. For this article, we will discuss span measurement.

The span measurement of teeth, W, is a measure over a number of teeth, *k*, made by means of a special tooth thickness micrometer. The value measured is the sum of normal tooth thickness on the base circle, sbn, and normal pitch, $P_{bn}(k-1)$. (See Figure 1).

Figure 1 details the span measurement of a spur gear. As such, the span measurement is on the outside of the teeth. For internal gears, the tooth profile is opposite to that of the external spur gear. As such, the measurement for an internal gear is between the inside of the tooth profiles. The calculations for both an external spur gear and an internal ring gear are detailed in Table 1.

Measuring helical gears can be done in either the normal plane or the transverse plane dependent on how the gears are cut. The formulas for measurements in the normal plane are detailed in Table 2, and those for measurements in the transverse plane are detailed in Table 3.

There is a requirement of a minimum face width to make a helical gear span measurement. Let b_{min} be the minimum value for face width. See Figure 2.

Then $b_{min} = W\sin\beta_b + \Delta b$

Where β_b is the helix angle at the base cylinder,

$\beta_b = \tan^{-1}(\tan\beta\cos\alpha_t)$

$\beta_b = \sin^{-1}(\sin\beta\cos\alpha_n)$

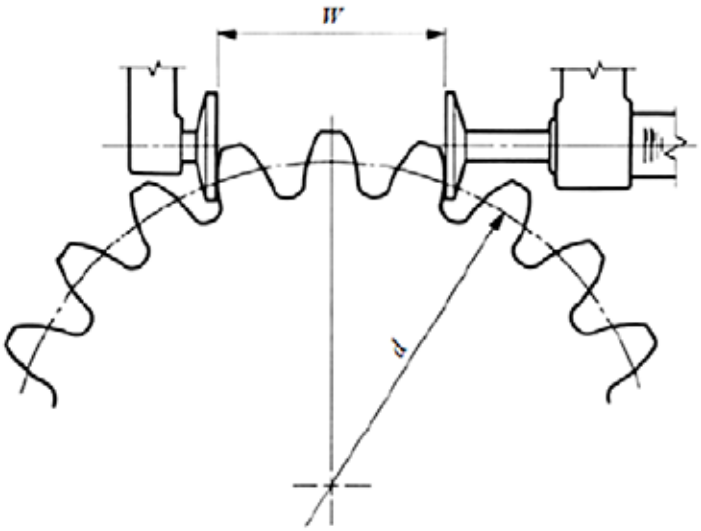

Figure 1: Span measurement over *k* teeth (spur gear).

These calculations resolve to show that a minimum value of $\Delta b > 3$mm is required in order to have a valid value for *W*.

Due to the tooth form of a bevel gear, whether it is a straight or spiral tooth, this span measurement technique cannot be used. The span measurement technique is also not employed for a worm wheel or a worm.

Using the above tables and formulas, you will be able to determine the proper span measurement for your spur gear, helical gear or internal ring gear. From these values and the measured values, you can determine the tooth thinning or backlash allowance cut into the gear.

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK\ (f) + 0.5$ NOTE 1 Select the nearest natural number of k_{th} as k	$m = 3$ $\alpha = 20°$ $z = 24$ $x = +0.4$ $k_{th} = 3.78787$ $k = 4$ $W = 32.8266$
2	Span measurement over k teeth	W	$m\cos\alpha\{\pi\ (k-0.5) + z\,\mathrm{inv}\,\alpha\} + 2xm\sin\alpha$	

NOTE: $K(f) = \frac{1}{\pi}\{\sec\alpha\sqrt{(1+2f)^2-\cos^2\alpha} - \mathrm{inv}\,\alpha - 2f\tan\alpha\}$ (1)

Where $f = \frac{x}{z}$

Table 1: Span measurement calculations for spur and internal gear teeth.

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No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK\ (f,\beta) + 0.5$ NOTE 2 Select the nearest natural number of k_{th} as k	$m_n = 3$, $\alpha_n = 20°$, $z = 24$ $\beta = 25°00'\ 00''$ $x_n = +0.4$ $\alpha_t = 21.88023°$ $k_{th} = 4.63009$ $k = 5$ $W = 42.0085$
2	Span measurement over k teeth	W	$m_n \cos\alpha_n \{\pi\ (k - 0.5) + z\ \mathrm{inv}\ \alpha_t\} + 2x_n m_n \sin\alpha_n$	

$$\text{NOTE}: \quad K(f,\beta) = \frac{1}{\pi}\left\{\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\,\alpha_t - 2f\tan\alpha_n\right\} \qquad (2)$$

$$\text{Where} \quad f = \frac{x_n}{z}$$

Table 2: Equations for the span measurement of normal system helical gears.

No.	Item	Symbol	Formula	Example
1	Span number of teeth	k	$k_{th} = zK\ (f,\beta) + 0.5$ NOTE 3 Select the nearest natural number of k_{th} as k	$m_t = 3$, $\alpha_t = 20°$, $z = 24$ $\beta = 22°30'\ 00''$ $x_t = +0.4$ $\alpha_n = 18.58597°$ $k_{th} = 4.31728$ $k = 4$ $W = 30.5910$
2	Span measurement over k teeth	W	$m_t \cos\beta \cos\alpha_n \{\pi\ (k - 0.5) + z\ \mathrm{inv}\ \alpha_t\} + 2x_t m_t \sin\alpha_n$	

$$\text{NOTE}: \quad K(f,\beta) = \frac{1}{\pi}\left\{\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\,\alpha_t - 2f\tan\alpha_n\right\} \qquad (3)$$

$$\text{where} \quad f = \frac{x_t}{z\cos\beta}$$

Table 3: Equations for span measurement of transverse system helical gears.

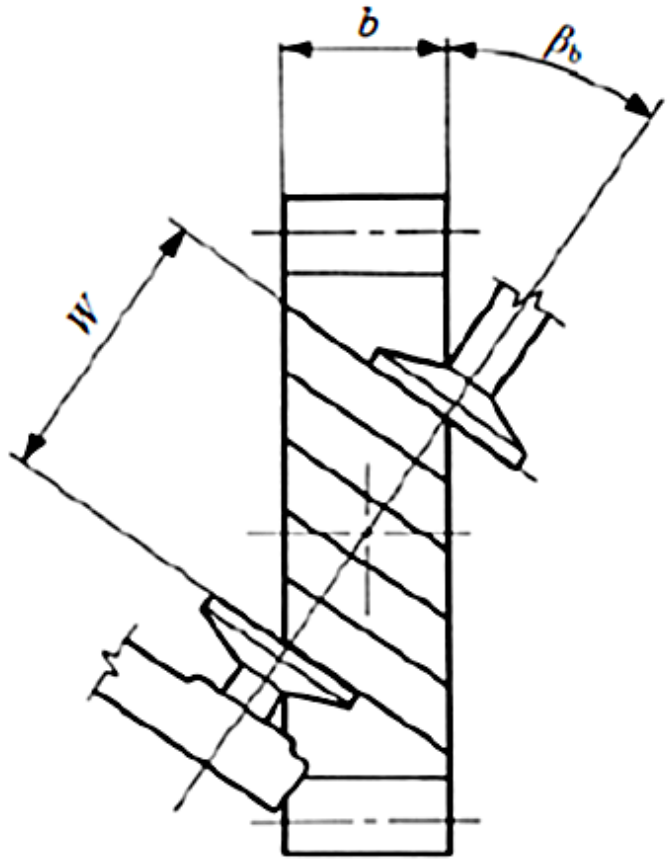

Figure 2: Face width of helical gear.