TOOTH TIPS

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Determining tooth thickness of various gear types – Part III

How to calculate the nominal values of over-pin or ball measurement of teeth for various types of gearing.

In order to determine the tooth size of a gear after taking into account the backlash allowance, you first must determine what the nominal tooth thickness should be. There are three methods for determining this value: chordal tooth thickness measurement, span measurement, and over-pin or ball measurement. For this article, we will discuss measurement over rollers, which is more commonly known as over-pin or ball measurement.

The over-pin measurement, *M*, is made over the outside of two pins that are inserted in diametrically opposite tooth spaces for even tooth number gears and as close as possible for odd tooth number gears. See Figure 1a for details.

In measuring a standard spur gear, the size of the pin must meet the condition that its surface should have a tangent point at the standard pitch circle. When measuring a profile shifted gear, the surface of the pin should have a tangent point at the $d + 2xm$ circle. Under the conditions mentioned earlier, Table 1 details the formulas that determine the diameter of the pin (ball) for the spur gear in Figure 1b.

An ideal diameter of pins when calculated from the equations of Table 1 may not be practical. So, in practice, you should select a standard pin diameter close to the ideal value. After the actual diameter of pin d_p is determined, the over-pin measurement M can be calculated from Table 2.

In Table 3 the calculated values for pin size under the conditions of module $m = 1$ and pressure angle $\alpha = 20°$ wherein the pin has the tangent point at $d + 2xm$ circle.

If you are measuring a straight tooth rack, the pin is ideally tangent with the tooth flank at the pitch line. The equations in Table 4 can, thus, be derived. In the case of a helical rack with module m and pressure angle α, in Table 4, can be substituted by normal module m_n , and normal pressure angle α_n , resulting in Table 5.

As shown in Figure 3, measuring an internal gear needs a proper pin that has its tangent point at $d + 2xm$ circle. The equations are in Table 6 for obtaining the ideal pin diameter. The equations for calculating the between-pin measurement, M, are in Table 7.

The Table 8 lists ideal pin diameters for standard and profile shifted internal gears under the conditions of module $m = 1$ and pressure angle $\alpha = 20°$, which makes the pin tangent to the reference circle $d + 2xm$.]

Another gear type to consider is the helical gear. The ideal pin that makes contact at the $d + 2x_n m_n$ reference circle of a helical gear can be obtained from the same above equations but with the teeth number z substituted by the equivalent (virtual) teeth number z_v.

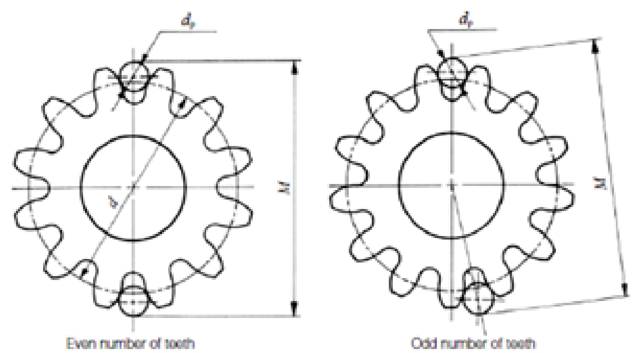

Figure 1a: Over-pin (ball) measurement.

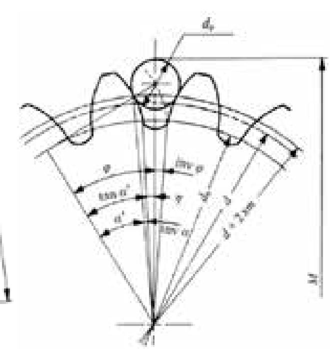

Figure 1b: Over-pins measurement of spur gear (far right).

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\frac{\pi}{2z} - \text{inv}\,\alpha\right) - \frac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 20$ $x = 0$ $\eta = 0.0636354$ $\alpha' = 20°$ $\varphi = 0.4276057$ $d'_p = 1.7245$
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\frac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	φ	$\tan\alpha' + \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\ (\text{inv}\,\varphi + \eta)$	

NOTE: The units of angles η and φ are radians.

Table 1: Equations for calculating ideal pin diameters.

No.	Item	Symbol	Formula	Example
1	Pin diameter	d_p	NOTE 1	$d_p = 1.7$ $\text{inv}\varphi = 0.0268197$ $\varphi = 24.1350°$ $M = 22.2941$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{zm\cos\alpha} - \frac{\pi}{2z} + \text{inv}\,\alpha + \frac{2x\tan\alpha}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\frac{zm\cos\alpha}{\cos\varphi} + d_p$ Odd teeth $\frac{zm\cos\alpha}{\cos\varphi}\cos\frac{90°}{z} + d_p$	

NOTE: The value of the ideal pin diameter from Table 1, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 2: Equations for over-pins measurement of spur gears.

Table 9 presents equations for deriving over-pin diameters. Table 10 presents equations for calculating over-pin measurements for helical gears in the normal system.

Tables 11 and 12 present equations for calculating the pin measurements for helical gears in the transverse (perpendicular to axis) system.

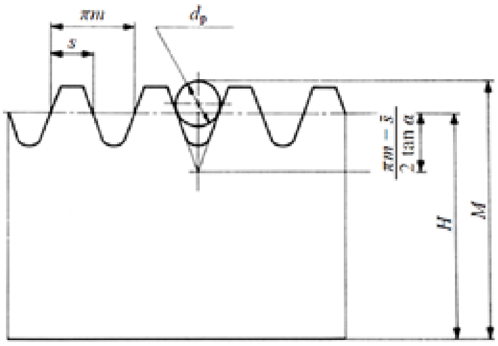

Figure 2: Over-pins measurement for a rack using a pin or a ball.

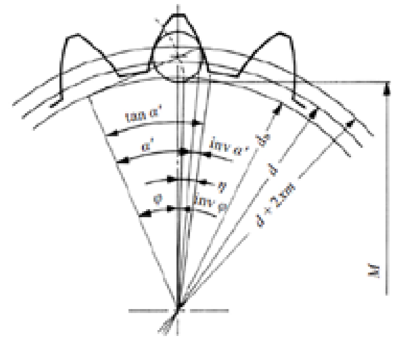

Figure 3: Between pin dimension of internal gears.

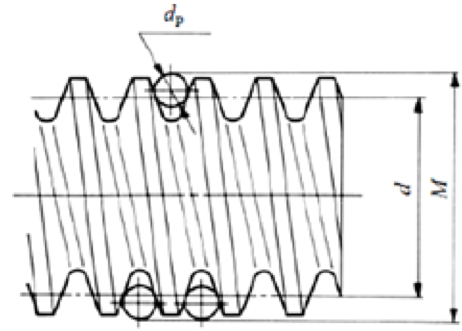

Figure 4: Three wire method of a worm.

$m = 1$、$\alpha = 20°$

No. of teeth z	Profile shift coefficient x							
	−0.4	−0.2	0	0.2	0.4	0.6	0.8	1.0
10		1.6347	1.7886	1.9979	2.2687	2.6079	3.0248	3.5315
20	1.6231	1.6599	1.7244	1.8149	1.9306	2.0718	2.2389	2.4329
30	1.6418	1.6649	1.7057	1.7632	1.8369	1.9267	2.0324	2.1542
40	1.6500	1.6669	1.6967	1.7389	1.7930	1.8589	1.9365	2.0257
50	1.6547	1.6680	1.6915	1.7247	1.7675	1.8196	1.8810	1.9515
60	1.6577	1.6687	1.6881	1.7155	1.7509	1.7940	1.8448	1.9032
70	1.6598	1.6692	1.6857	1.7090	1.7391	1.7759	1.8193	1.8691
80	1.6613	1.6695	1.6839	1.7042	1.7304	1.7625	1.8003	1.8438
90	1.6625	1.6698	1.6825	1.7005	1.7237	1.7521	1.7857	1.8242
100	1.6635	1.6700	1.6814	1.6975	1.7184	1.7439	1.7740	1.8087
110	1.6642	1.6701	1.6805	1.6951	1.7140	1.7372	1.7645	1.7960
120	1.6649	1.6703	1.6797	1.6931	1.7104	1.7316	1.7567	1.7855
130	1.6654	1.6704	1.6791	1.6914	1.7074	1.7269	1.7500	1.7766
140	1.6659	1.6705	1.6785	1.6900	1.7048	1.7229	1.7443	1.7690
150	1.6663	1.6706	1.6781	1.6887	1.7025	1.7194	1.7394	1.7625
160	1.6666	1.6706	1.6777	1.6876	1.7006	1.7164	1.7351	1.7567
170	1.6669	1.6707	1.6773	1.6867	1.6988	1.7137	1.7314	1.7517
180	1.6672	1.6707	1.6770	1.6858	1.6973	1.7114	1.7280	1.7472
190	1.6674	1.6708	1.6767	1.6851	1.6959	1.7093	1.7250	1.7432
200	1.6676	1.6708	1.6764	1.6844	1.6947	1.7074	1.7223	1.7396

Table 3: The size of pin which has the tangent point at d = 2xm circle for spur gears.

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m - s}{\cos\alpha}$	$m = 1$ $\alpha = 20°$ $s = 1.5708$ $d'_p = 1.6716$ $d_p = 1.7$ $H = 14.0000$ $M = 15.1774$
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m - s}{2\tan\alpha} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha}\right)$	

Table 4: Equations for over-pins measurement of straight racks.

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\dfrac{\pi m_n - s}{\cos\alpha_n}$	$m_n = 1$ $\alpha_n = 20°$、$\beta = 15°$ $s = 1.5708$ $d'_p = 1.6716$ $d_p = 1.7$ $H = 14.0000$ $M = 15.1774$
2	Measurement over pin (ball)	M	$H - \dfrac{\pi m_n - s}{2\tan\alpha_n} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha_n}\right)$	

Table 5: Equations for over-pins measurement of helical racks.

No.	Item	Symbol	Formula	Example
1	Spacewidth half angle	η	$\left(\dfrac{\pi}{2z} + \mathrm{inv}\,\alpha\right) + \dfrac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 40$ $x = 0$ $\eta = 0.054174$ $\alpha' = 20°$ $\varphi = 0.309796$ $d'_p = 1.6489$
2	Pressure angle at the point pin is tangent to tooth surface	α'	$\cos^{-1}\left\{\dfrac{zm\cos\alpha}{(z+2x)m}\right\}$	
3	Pressure angle at pin center	φ	$\tan\alpha' - \eta$	
4	Ideal pin diameter	d'_p	$zm\cos\alpha\,(\eta - \mathrm{inv}\,\varphi)$	

NOTE: The units of angles η, φ are radians.

Table 6: Equations for calculating pin diameter for internal gears.

As noted in Figure 4, worms can be measured using the three-wire method. The tooth profile of type III worms, which are the most popular, are cut by standard cutters with a pressure angle $\alpha_0 = 20°$. This results in the normal pressure angle of the worm being a bit smaller than 20°. Equation 1 shows how to calculate the normal pressure angle of a type III worm in the AGMA system.

$$\alpha_n = \alpha_0 - \frac{90}{z_1}\,\frac{r}{r_0\cos^2\gamma + r}\,\sin^3\gamma$$

Where
r: Worm reference radius
r_0: Cutter radius
z_1: Number of threads
γ: Lead angle of worm

The exact equation for measuring type III worms using the three-wire method is not only difficult to comprehend but also hard to calculate precisely. As such, there are two approximate calculation methods that you can use:

a) Regard the tooth profile of the worm as a straight tooth profile of a rack and apply its equations. Using this system, the three-wire method of a worm can be calculated as detailed in Table 13.

These equations presume the worm lead angle to be very small and can be neglected. Of course, as the lead angle gets larger, the equations' error gets correspondingly larger. If the lead angle is considered as a factor, the proper equations are detailed in Table 14.

b) Consider a worm to be a helical gear. This means applying the equations for calculating over pins measurement of a helical gear to the case of three wire method for a worm. Because the tooth profile of Type III worm is not an involute curve, this method yields an approximation. However, the accuracy is quite adequate in practice.

Tables 15 and 16 contain equations based on the axial system. Tables 17 and 18 are based on the normal system.

Tables 17 and 18 show the calculations of a worm in the normal module system. Basically, the normal module system and the axial module system have the same form of equations. Only the notations of module make them different.

Due to the tooth form of bevel gearing, whether it is straight or spiral tooth, this measurement over pins method is not possible and cannot be used.

Using these tables and formulas, you will be able to determine the proper over-rollers measurement for your spur gear, helical gear, internal ring gear, gear rack, or worm. From these calculated values and the actual measured values, you can determine the tooth thinning or backlash allowance cut into the gear.

ABOUT THE AUTHOR

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No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	see NOTE 1	$d_p = 1.7$ $\text{inv}\varphi = 0.0089467$ $\varphi = 16.9521°$ $M = 37.5951$
2	Involute function φ	$\text{inv}\,\varphi$	$\left(\frac{\pi}{2z} + \text{inv}\,\alpha\right) - \frac{d_p}{zm\cos\alpha} + \frac{2x\tan\alpha}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Between pins measurement	M	Even teeth $\frac{zm\cos\alpha}{\cos\varphi} - d_p$	
			Odd teeth $\frac{zm\cos\alpha}{\cos\varphi}\cos\frac{90°}{z} - d_p$	

NOTE: First, calculate the ideal pin diameter. Then, choose the nearest practical actual pin size.

Table 7: Equations for between pin measurement of internal gears (above).

$m = 1,\ \alpha = 20°$

No. of teeth z	Profile shift coefficient x							
	− 0.4	− 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.4789	1.5936	1.6758	1.7283	1.7519	1.7460	1.7092
20	1.4687	1.5604	1.6284	1.6759	1.7047	1.7154	1.7084	1.6837
30	1.5309	1.5942	1.6418	1.6751	1.6949	1.7016	1.6956	1.6771
40	1.5640	1.6123	1.6489	1.6745	1.6895	1.6944	1.6893	1.6744
50	1.5845	1.6236	1.6532	1.6740	1.6862	1.6900	1.6856	1.6732
60	1.5985	1.6312	1.6562	1.6737	1.6839	1.6870	1.6832	1.6725
70	1.6086	1.6368	1.6583	1.6734	1.6822	1.6849	1.6815	1.6721
80	1.6162	1.6410	1.6600	1.6732	1.6810	1.6833	1.6802	1.6718
90	1.6222	1.6443	1.6612	1.6731	1.6800	1.6820	1.6792	1.6717
100	1.6270	1.6470	1.6622	1.6729	1.6792	1.6810	1.6784	1.6715
110	1.6310	1.6492	1.6631	1.6728	1.6785	1.6801	1.6778	1.6715
120	1.6343	1.6510	1.6638	1.6727	1.6779	1.6794	1.6772	1.6714
130	1.6371	1.6525	1.6644	1.6727	1.6775	1.6788	1.6768	1.6714
140	1.6395	1.6539	1.6649	1.6726	1.6771	1.6783	1.6764	1.6714
150	1.6416	1.6550	1.6653	1.6725	1.6767	1.6779	1.6761	1.6713
160	1.6435	1.6561	1.6657	1.6725	1.6764	1.6775	1.6758	1.6713
170	1.6451	1.6570	1.6661	1.6724	1.6761	1.6771	1.6755	1.6713
180	1.6466	1.6578	1.6664	1.6724	1.6759	1.6768	1.6753	1.6713
190	1.6479	1.6585	1.6666	1.6723	1.6757	1.6766	1.6751	1.6713
200	1.6490	1.6591	1.6669	1.6723	1.6755	1.6763	1.6749	1.6713

Table 8: The size of pin that is tangent at reference circle d + 2xm for internal gears (left).

Table 9: Equations for calculating pin diameter for helical gears in the normal system. (below)

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	$m_n = 1$ $\alpha_n = 20°$ $z = 20$ $\beta = 15°00'00''$ $x_n = +0.4$ $z_v = 22.19211$ $\eta_v = 0.0427566$ $\alpha'_v = 24.90647°$ $\varphi_v = 0.507078$ $d'_p = 1.9020$
2	Spacewidth half angle	η_v	$\frac{\pi}{2z_v} - \text{inv}\,\alpha_n - \frac{2x_n\tan\alpha_n}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\frac{z_v\cos\alpha_n}{z_v + 2x_n}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n\cos\alpha_n\ (\text{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 10: Equations for calculating over-pins measurement for helical gears in the normal system.

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 2$ $\alpha_t = 20.646896°$ $\text{inv}\,\varphi = 0.058890$ $\varphi = 30.8534°$ $M = 24.5696$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_n z\cos\alpha_n} - \frac{\pi}{2z} + \text{inv}\,\alpha_t + \frac{2x_n\tan\alpha_n}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even Teeth $\frac{zm_n\cos\alpha_t}{\cos\beta\cos\varphi} + d_p$	
			Odd Teeth $\frac{zm_n\cos\alpha_t}{\cos\beta\cos\varphi}\cos\frac{90°}{z} + d_p$	

NOTE 1: The ideal pin diameter of Table 9, or its approximate value, is entered as the actual diameter of d_p.

Table 11: Equations for calculating pin diameter for helical gears in the transverse system.

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	$m_t = 3$ $\alpha_t = 20°$ $z = 36$ $\beta = 33°33'26.3''$ $\alpha_n = 16.87300°$ $x_t = +0.2$ $z_v = 62.20800$ $\eta_v = 0.014091$ $\alpha'_v = 18.26390$ $\varphi_v = 0.34411$ $\text{inv}\,\varphi_v = 0.014258$ $d'_p = 4.2190$
2	Spacewidth half angle	η_v	$\frac{\pi}{2z_v} - \text{inv}\,\alpha_n - \frac{2x_t\tan\alpha_t}{z_v}$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\frac{z_v\cos\alpha_n}{z_v + 2\frac{x_t}{\cos\beta}}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_t\cos\beta\cos\alpha_n\ (\text{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 12: Equations for calculating over-pins measurement for helical gears in the transverse system.

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 4.5$ $\text{inv}\,\varphi = 0.027564$ $\varphi = 24.3453°$ $M = 115.892$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_t z\cos\beta\cos\alpha_n} - \frac{\pi}{2z} + \text{inv}\,\alpha_t + \frac{2x_t\tan\alpha_t}{z}$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Measurement over pin (ball)	M	Even teeth $\frac{z m_t\cos\alpha_t}{\cos\varphi} + d_p$	
			Odd teeth $\frac{z m_t\cos\alpha_t}{\cos\varphi}\cos\frac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of Table 11, or its approximate value is applied as the actual diameter of pin dp here.

Table 13: Equations for three wire method of worm measurement.

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\frac{\pi m_x}{2\cos\alpha_x}$	$m_x = 2$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $\alpha_x = 20.03827°$ $d'_p = 3.3440$ $d_p = 3.3$ $M = 35.3173$
2	Three wire measurement	M	$d_1 - \frac{\pi m_x}{2\tan\alpha_x} + d_p\left(1 + \frac{1}{\sin\alpha_x}\right)$	

Table 14: Equations for three wire method of worm measurement.

No.	Item	Symbol	Formula	Example
1	Ideal pin diameter	d'_p	$\frac{\pi m_n}{2\cos\alpha_n}$	$m_x = 2$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $m_n = 1.99585$ $d'_p = 3.3363$ $d_p = 3.3$ $M = 35.3344$
2	Three wire measurement	M	$d_1 - \frac{\pi m_n}{2\tan\alpha_n} + d_p\left(1 + \frac{1}{\sin\alpha_n}\right) - \frac{(d_p\cos\alpha_n\sin\gamma)^2}{2d_1}$	

Table 15: Equations for calculating pin diameter for worms in the axial system.

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\frac{z_1}{\cos^3(90° - \gamma)}$	$m_x = 2$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $z_v = 3747.1491$ $\eta_v = -0.014485$ $\alpha'_v = 20°$ $\varphi_v = 0.349485$ $\text{inv}\,\varphi_v = 0.014960$ $d'_p = 3.3382$
2	Spacewidth half angle	η_v	$\frac{\pi}{2z_v} - \text{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\frac{z_v\cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_x\cos\gamma\cos\alpha_n\,(\text{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 16: Equations for three wire method for worms in the axial system.

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 3.3$ $\alpha_t = 76.96878°$ $\text{inv}\,\alpha_t = 4.257549$ $\text{inv}\,\varphi = 4.446297$ $\varphi = 80.2959°$ $M = 35.3345$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_x z_1\cos\gamma\cos\alpha_n} - \frac{\pi}{2z_1} + \text{inv}\,\alpha_t$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Three wire measurement	M	$\frac{z_1 m_x\cos\alpha_t}{\tan\gamma\cos\varphi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 15, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\sin\gamma}\right)$

Table 17: Equations for calculating pin diameter for worms in the normal system.

No.	Item	Symbol	Formula	Example
1	Number of teeth of an equivalent spur gear	z_v	$\frac{z_1}{\cos^3(90° - \gamma)}$	$m_n = 2.5$ $\alpha_n = 20°$ $z_1 = 1$ $d_1 = 37$ $\gamma = 3.874288°$ $z_v = 3241.792$ $\eta_v = -0.014420$ $\alpha'_v = 20°$ $\varphi_v = 0.349550$ $\text{inv}\,\varphi_v = 0.0149687$ $d'_p = 4.1785$
2	Spacewidth half angle	η_v	$\frac{\pi}{2z_v} - \text{inv}\,\alpha_n$	
3	Pressure angle at the point pin is tangent to tooth surface	α'_v	$\cos^{-1}\left(\frac{z_v\cos\alpha_n}{z_v}\right)$	
4	Pressure angle at pin center	φ_v	$\tan\alpha'_v + \eta_v$	
5	Ideal pin diameter	d'_p	$z_v m_n\cos\alpha_n\,(\text{inv}\,\varphi_v + \eta_v)$	

NOTE: The units of angles η_v and φ_v are radians.

Table 18: Equations for three wire method for worms in the normal system.

No.	Item	Symbol	Formula	Example
1	Pin (ball) diameter	d_p	See NOTE 1	$d_p = 4.2$ $\alpha_t = 79.48331°$ $\text{inv}\,\alpha_t = 3.999514$ $\text{inv}\,\varphi = 4.216536$ $\varphi = 79.8947°$ $M = 42.6897$
2	Involute function φ	$\text{inv}\,\varphi$	$\frac{d_p}{m_n z_1\cos\alpha_n} - \frac{\pi}{2z_1} + \text{inv}\,\alpha_t$	
3	Pressure angle at pin center	φ	Find from involute function table	
4	Three wire measurement	M	$\frac{z_1 m_n\cos\alpha_t}{\sin\gamma\cos\varphi} + d_p$	

NOTE 1. The value of ideal pin diameter from Table 17, or its approximate value, is to be used as the actual pin diameter, d_p.

NOTE 2. $\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\sin\gamma}\right)$