



Addendum modification and undercut

How to calculate the coefficient of profile shift to achieve addendum modification to eliminate undercut.

When you walk onto the sales floor in any furniture store, you are greeted by a smiling, courteous, friendly sales associate. Their job is to steer you in the direction of the product that suits your needs. These needs could be budgetary, they could be functionality, they could be style, or they could be a combination of all three. The one thing that they dread doing is offering you a price that is lower than you planned to spend, as this will undercut their commission. In gearing, undercutting is also a condition that everyone wishes to avoid.

Involute gears can be easily generated by rack type cutters. A hob is, in effect, a rack cutter. Gear generation can also be produced with a gear shaper or planer machine. Figure 1 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear. Gear shaper machines, with pinion cutters, can also be used to generate involute gears. Gear shapers can not only generate external gears but also generate internal gear teeth.

When cutting a spur pinion like the gear shown in Figure 1, undercutting occurs if you cut deeper than the interfering point, I. Undercutting is a phenomenon that occurs when some part of tooth dedendum is unexpectedly cut by the edge of the generating tool. In order to create the conditions for no undercutting of a standard spur gear, the following conditions must be met:

At the maximum addendum:

$$m \leq \frac{mz}{2} \sin^2 \alpha$$

Where m = the module
z = number of teeth
 α = the pressure angle

And the minimum number of teeth is:

$$z = \frac{2}{\sin^2 \alpha}$$

When the pressure angle is 20 degrees, the minimum number of teeth for a spur gear to be free of undercutting is 17. However, gears with 16 teeth or less can be usable if their strength and contact ratio still meet the design requirements.

If undercutting of the pinion does affect the gear design requirements, then profile shifting or addendum modification techniques can be employed. As shown in Figure 1, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut. To prevent this undercut, a positive correction must be introduced. This positive correction or profile shift, as shown in Figure 2, can prevent undercut.

Undercutting will get worse if a negative correction is applied as detailed in Figure 3.

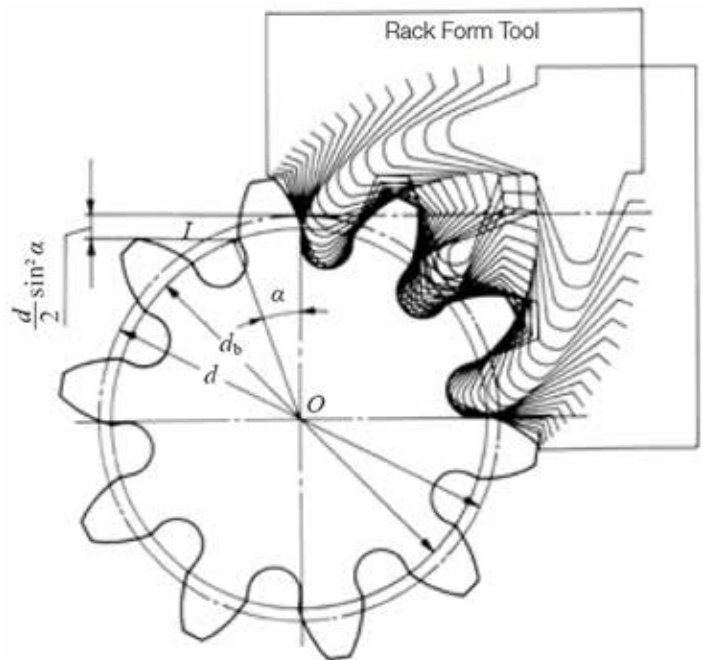


Figure 1: Generation of a Standard Spur Gear.
($\alpha = 20^\circ$, $z = 10$, $x = 0$)

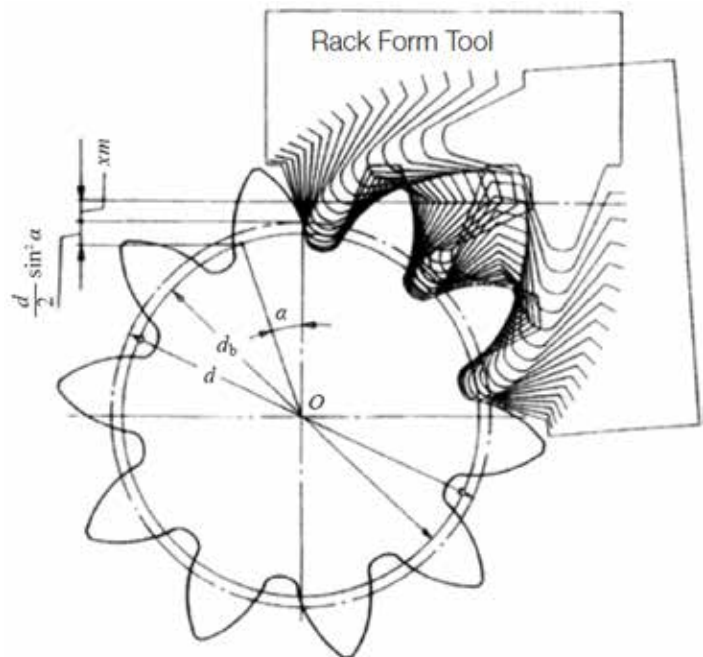


Figure 2: Generation of Positive Shifted Spur Gear.
($\alpha = 20^\circ$, $z = 10$, $x = 0.5$)

The extra feed of gear cutter (xm) in Figures 2 and 3 is the amount of shift or correction. And x is the profile shift coefficient.

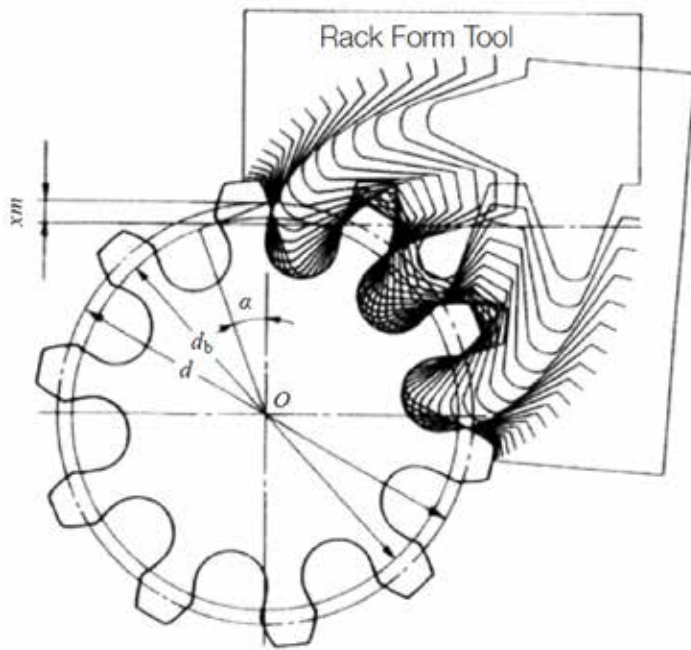


Figure 3: Generation of Negative Shifted Spur Gear.
 ($\alpha = 20^\circ$, $z = 10$, $x = -0.5$)

The condition to prevent undercut in a spur gear is:

$$m - xm \leq \frac{zm}{2} \sin^2 \alpha$$

The number of teeth without undercut will be:

$$z = \frac{2(1-x)}{\sin^2 \alpha}$$

The profile shift coefficient without undercut is:

$$x = 1 - \frac{z}{2} \sin^2 \alpha$$

Profile shift is not merely used to prevent undercut; it can also be used to adjust the center distance between two gears. If a positive correction is applied, such as to prevent undercut on a pinion, the tooth tip becomes pointed.

Table 1 details the theoretical calculations to determine the top land thickness, s_a , for a module 2, 16 tooth spur gear with a profile shift of 0.3.

Using the tables and formulas shown here, you will be able to determine the proper number of teeth to prevent undercutting and you can calculate the tooth tip top land thickness. ☞

No.	Item	Symbols	Symbol	Formula	Example
1	Module	m	mm	Set Value	2
2	Pressure angle	α	Degree		20
3	No. of Teeth	z	-		16
4	Profile Shift Coefficient	x	-		0.3
5	Reference Diameter	d	mm	zm	32
6	Base Diameter	d_b		$d \cos \alpha$	30.07016
7	Tip Diameter	d_a		$d + 2m(1+x)$	37.2
8	Tip Pressure Angle	α_a	Degree	$\cos^{-1} \frac{d_b}{d_a}$	36.06616
9	Involute α	$\text{inv} \alpha$	Radian	$\tan \alpha - \alpha$	0.014904
10	Involute α_a	$\text{inv} \alpha_a$		$\tan \alpha_a - \alpha_a$	0.098835
11	Tip Tooth Thickness Half Angle	ψ_a		$\frac{\pi}{2z} + \frac{2x \tan \alpha}{z} + (\text{inv} \alpha - \text{inv} \alpha_a)$	0.027893
12	Crest Width	s_a	mm	$\psi_a d_a$	1.03762

Table 1: Calculations of Top Land Thickness (Crest Width)